ON THE REFLECTION OF SOUND WAVES FROM STRATIFIED TWO-COMPONENT MEDIA

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The dynamics of multi-component media has an important application in the analysis of building foundations, in seismology, in problems of sound-proofing, in studies of the motion of pulp, aerated oil, etc.

This theory was developed by Frenkel' [1], Leibenzon [2], Biot [3, 4], Rakhmatulin [5], Zwikker and Kosten [6], and others.

Reference [7] treats the propagation of elastic waves in an isotropic two-component medium, one component of which is ideally elastic, while the other is a viscous compressible fluid. It has been shown that in this case the equations of Biot can be considered the most general equations of motion.

On the basis of Biot's equations, this paper is concerned with the solution of the problem of the propagation of plane sound waves in the above-mentioned two-component medium with a stratified structure. General expressions for the reflection and transmission coefficients are obtained for an arbitrary number of strata. The particular case of a single stratum is studied in greater detail.

The two-component medium is further treated as a porous medium with an elastic skeleton [matrix] and pores which are filled with a viscous compressible fluid.

1. Fundamental equations. The relations between the stress and strain tensors for isotropic porous media, as established in [3], can be written as

$$P_{ik} = \lambda\theta\delta_{ik} + 2\mu u_{ik} + Q\varepsilon\delta_{ik}, \qquad S = Q\theta + R\varepsilon$$
$$u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right), \qquad \theta = \operatorname{div} \mathbf{u}, \qquad \varepsilon = \operatorname{div} \mathbf{v} \tag{1.1}$$

Here P_{ik} is the stress tensor in the elastic skeleton; S is a force acting on the fluid referred to a unit cross-sectional area of the porous medium; λ, μ, Q, and R are moduli characterizing the elasticity of the porous medium; u_{ik} is the strain tensor of the skeleton; u and v are vectors of the mean displacements of the skeleton and the fluid at a given point of the medium; δ_{ik} is the Kronecker delta, equal to unity for $i = k$, and equal to zero for $i \neq k$.

Biot's equations have the form

$$\rho_{11}\frac{\partial^2 u_i}{\partial t^2} + \rho_{12}\frac{\partial^2 v_i}{\partial t^2} + b\frac{\partial}{\partial t}(u_i - v_i) = \frac{\partial P_{ik}}{\partial x_k}, \quad \rho_{11} = (1-m)\rho_s - \rho_{12} \tag{1.2}$$

$$\rho_{12}\frac{\partial^2 u_i}{\partial t^2} + \rho_{22}\frac{\partial^2 v_i}{\partial t^2} + b\frac{\partial}{\partial t}(v_i - u_i) = \frac{\partial S}{\partial x_i}, \quad \rho_{22} = m\rho_f - \rho_{12}\left(b = \frac{\mu m^2}{k}\right)$$

Here $\rho_{12} < 0$ is the dynamic coupling coefficient between the skeleton and the fluid; ρ_s is the density of the skeleton; ρ_f is the density of the fluid; k is the permeability coefficient, proportional to the porosity and the square of the pore diameter. Let us break up the displacement vectors into their irrotational and solenoidal components

$$\mathbf{u} = \mathbf{u}_l + \mathbf{u}_t, \quad \operatorname{rot}\mathbf{u}_l = 0, \quad \operatorname{div}\mathbf{u}_t = 0$$
$$\mathbf{v} = \mathbf{v}_l + \mathbf{v}_t, \quad \operatorname{rot}\mathbf{v}_l = 0, \quad \operatorname{div}\mathbf{v}_t = 0 \tag{1.3}$$

When taking Equation (1.2) into account, (1.1) and (1.3) can be reduced to the following system:

$$\rho_{11}\frac{\partial^2 \mathbf{u}_l}{\partial t^2} + \rho_{12}\frac{\partial^2 \mathbf{v}_l}{\partial t^2} + b\frac{\partial}{\partial t}(\mathbf{u}_l - \mathbf{v}_l) = (\lambda + 2\mu)\nabla^2\mathbf{u}_l + Q\nabla^2\mathbf{v}_l$$
$$\rho_{12}\frac{\partial^2 \mathbf{u}_l}{\partial t^2} + \rho_{22}\frac{\partial^2 \mathbf{v}_l}{\partial t^2} + b\frac{\partial}{\partial t}(\mathbf{v}_l - \mathbf{u}_l) = Q\nabla^2\mathbf{u}_l + R\nabla^2\mathbf{v}_l$$
$$\rho_{11}\frac{\partial^2 \mathbf{u}_t}{\partial t^2} + \rho_{12}\frac{\partial^2 \mathbf{v}_t}{\partial t^2} + b\frac{\partial}{\partial t}(\mathbf{u}_t - \mathbf{v}_t) = \mu\nabla^2\mathbf{u}_t \tag{1.4}$$
$$\rho_{12}\frac{\partial^2 \mathbf{u}_t}{\partial t^2} + \rho_{22}\frac{\partial^2 \mathbf{v}_t}{\partial t^2} + b\frac{\partial}{\partial t}(\mathbf{v}_t - \mathbf{u}_t) = 0$$

In the case of monochromatic waves with a frequency ω, the first two equations of (1.4), with the aid of the linear transformations

$$\mathbf{u}_l = \mathbf{u}_1 + \mathbf{u}_2, \quad \mathbf{v}_l = M_1\mathbf{u}_1 + M_2\mathbf{u}_2 \tag{1.5}$$

and after the introduction of the notation

$$\gamma_{11} = \frac{\rho_{11}}{\rho}, \quad \gamma_{12} = \frac{\rho_{12}}{\rho}, \quad \gamma_{22} = \frac{\rho_{22}}{\rho}, \quad \rho = \rho_{11} + \rho_{22} + 2\rho_{12}$$
$$\sigma_{11} = \frac{\lambda + 2\mu}{H}, \quad \sigma_{12} = \frac{Q}{H}, \quad \sigma_{22} = \frac{R}{H}, \quad H = \lambda + 2\mu + R + 2Q, \quad c^2 = \frac{H}{\rho} \tag{1.6}$$

become

$$\nabla^2 \mathbf{u}_1 + k_1^2 \mathbf{u}_1 = 0, \qquad \nabla^2 \mathbf{u}_2 + k_2^2 \mathbf{u}_2 = 0 \qquad \left(k_1^2 = \zeta_1 \left(\frac{\omega}{c}\right)^2, \ k_2^2 = \zeta_2 \left(\frac{\omega}{c}\right)^2\right)$$

Here ζ_1 and ζ_2 are the roots of the quadratic equation

$$(\sigma_{11}\sigma_{22} - \sigma_{12}^2)\,\zeta^2 - (\sigma_{11}\gamma_{22} + \sigma_{22}\gamma_{11} - 2\sigma_{12}\gamma_{12})\,\zeta + \gamma_{11}\gamma_{22} - \gamma_{12}^2 + \\ + \frac{i\beta}{\rho\omega}(\zeta - 1) = 0$$

The transformation coefficients in (1.5) are determined by the formulas

$$M_1 = \frac{-\gamma_{12} + \zeta_1\sigma_{12} + i\gamma}{\gamma_{22} - \zeta_1\sigma_{22} + i\gamma}, \qquad M_2 = \frac{-\gamma_{12} + \zeta_2\sigma_{12} + i\gamma}{\gamma_{22} - \zeta_2\sigma_{22} + i\gamma}, \qquad \gamma = \frac{b}{\rho\omega}$$

Equations (1.6) describe the propagation of longitudinal waves of the first and second type.

The second two equations of (1.4), which describe the propagation of the transverse wave, reduce to the form

$$v_t = M_t \mathbf{u}_t, \qquad \nabla^2 \mathbf{u}_t^a + k_t^2 \mathbf{u}_t = 0 \tag{1.7}$$

$$M_t = \frac{-\gamma_{12} + i\gamma}{\gamma_{22} + i\gamma}, \qquad k_t^2 = \frac{\rho_1 + M_t\rho_2}{\mu}\,\omega^2, \quad \rho_1 = \rho_{11} + \rho_{12} \qquad \rho_2 = \rho_{22} + \rho_{12}$$

Under the conditions of $\gamma \gg 1$, which corresponds to the case of a low frequency, it can be easily shown [7] that the damping coefficients of the first type of the longitudinal and the transverse waves are proportional to the square of the frequency, while those of the longitudinal waves of the second type are proportional to the square root of the frequency. This implies that the longitudinal wave of the second type disappears for all practical purposes.

If $\gamma \ll 1$, one can neglect the effect of viscosity. One should keep in mind, however, that ω must remain smaller than that frequency at which the wavelength is comparable to the dimensions of the pores.

2. Reflection and transmission coefficients for an arbitrary number of strata. The method of determining the reflection and transmission coefficients will be based on the use of recurrence formulas, which relate the wave amplitudes in neighboring strata [8,9].

Let us consider an arbitrary layer n. We denote its thickness by d and choose a coordinate system as shown in Fig. 1.

Because of the reflections from the boundaries, there will exist in the studied layer a system of transverse and both types of longitudinal waves, propagating in the positive and negative y-direction. The expressions for the potential of the longitudinal and transverse waves in

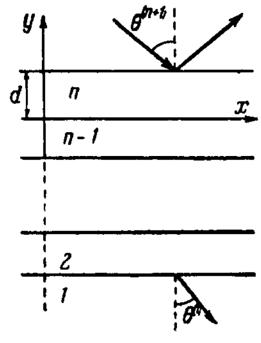

Fig. 1.

the layer can be then written in the form

$$\varphi_1 = (\varphi_1' e^{i\alpha y} + \varphi_1'' e^{-i\alpha y})\, e^{i(\sigma x - \omega t)} \qquad \alpha = \sqrt{k_1^2 - \sigma^2}$$
$$\varphi_2 = (\varphi_2' e^{i\beta y} + \varphi_2'' e^{-i\beta y})\, e^{i(\sigma x - \omega t)}, \qquad \beta = \sqrt{k_2^2 - \sigma^2}$$
$$\psi = (\psi' e^{i\delta y} + \psi'' e^{-i\delta y})\, e^{i(\sigma x - \omega t)}, \qquad \delta = \sqrt{k_t^2 - \sigma^2}$$

Here σ is the component of the wave vector along the x-axis, which is equal for all types of waves and all layers.

The velocity components of the skeleton particles $\partial u_x/\partial t$, $\partial u_y/\partial t$ and of the fluid $\partial v_y/\partial t$ at any point of a layer are found by means of the formulas.

$$\frac{\partial u_x}{\partial t} = \frac{\partial \varphi_1}{\partial x} + \frac{\partial \varphi_2}{\partial x} - \frac{\partial \psi}{\partial y}, \quad \frac{\partial u_y}{\partial t} = \frac{\partial \varphi_1}{\partial y} + \frac{\partial \varphi_2}{\partial y} + \frac{\partial \psi}{\partial x}, \frac{\partial v_y}{\partial t} = M_1 \frac{\partial \varphi_1}{\partial y} + M_2 \frac{\partial \varphi_2}{\partial y} + M_t \frac{\partial \varphi}{\partial x}$$

which follow from (1.3), (1.5), and (1.7).

The stress tensor components in the skeleton P_{yy}, P_{xy} and in the fluid S are determined from the relations (1.1). There, one should let

$$u_x = \frac{i}{\omega}\frac{\partial u_x}{\partial t}, \qquad u_y = \frac{i}{\omega}\frac{\partial u_y}{\partial t}, \qquad v_x = \frac{i}{\omega}\frac{\partial v_x}{\partial t}, \qquad v_y = \frac{i}{\omega}\frac{\partial v_y}{\partial t}$$

Let us denote by $G(n)$ a column matrix, whose elements, from top to bottom, will be the values of these quantities

$$\frac{\partial u_x}{\partial t}, \quad \frac{\partial u_y}{\partial t}, \quad P_{yy}, \quad \frac{1}{2\mu} P_{xy}, \quad \frac{1}{Q + RM_2} S, \quad j = (1 - m)\frac{\partial u_y}{\partial t} + m\frac{\partial v_y}{\partial t} \tag{2.1}$$

given at the upper boundary of the nth layer (at $y = d$). The results of the computation of these quantities can be written down in the form of a matrix equation

$$G(n) = A\Phi \tag{2.2}$$

Here Φ is a column matrix with the following elements (top to bottom):

$$\varphi_1' + \varphi_1'', \quad \varphi_1' - \varphi_1'', \quad \varphi_2' + \varphi_2'', \quad \varphi_2' - \varphi_2'', \quad \psi' - \psi'', \quad \psi' + \psi''$$

The elements of A, a quadratic matrix of the sixth order, can be, if necessary, written out easily.

If in (2.2) d is replaced by zero, we will obtain the values of the quantities (2.1) at the lower boundary of the nth layer. Since all these quantities remain unchanged as they cross over a boundary, they will also have the same values at the upper boundary of the $(n-1)$st layer. Thus, we have

$$G(n-1) = A_0\Phi, \qquad \text{or} \qquad \Phi = A_0^{-1}G(n-1) \tag{2.3}$$

Substitution of (2.3) into (2.2) yields a recurrence relation which relates the values of the quantities of (2.1) in neighboring layers:

$$G(n) = CG(n-1) \tag{2.4}$$

Here C will be the product of matrix A and matrix A_0^{-1}. If the quantities of (2.1) are given at the boundary between the first and the second medium, one can find them also at the boundary between the nth and the $(n+1)$st layers by means of a successive application of Formula (2.4):

$$G(n) = \Pi G(1) \tag{2.5}$$

Here Π is the product of the C-matrices for all layers.

Assume that the first and the $(n+1)$st media are fluid.

The conditions at the boundary between the nth and the $(n+1)$st medium have the form

$$P_{yy}^{(n)} = -(1-m^{(n)})P^{(n+1)}, \quad S^{(n)} = -m^{(n)}P^{(n+1)}, \quad P_{xy}^{(n)} = 0, \quad j^{(n)} = \frac{\partial v_y^{(n+1)}}{\partial t} \tag{2.6}$$

(The upper index denotes the number of the layer to which the given quantity is referred). The first two conditions consist of the fact that the skeleton and the fluid in the pores of the nth layer are under the same external pressure $P^{(n+1)}$. The third condition expresses the absence of tangential stresses, while the fourth condition denotes the continuity of fluid flow across the boundary.

After writing out the equations for $P_{yy}^{(n)}$, $P_{xy}^{(n)}$, $S^{(n)}$, $j^{(n)}$ and then eliminating from them the derivatives $\partial u_x^{(1)}/\partial t$, $\partial u_y^{(1)}/\partial t$ and considering that $P_{xy}^{(n)} = P_{xy}^{(1)} = 0$, $j^{(1)} = \partial v_y^{(1)}/\partial t$, we obtain

$$\begin{aligned} P_{yy}^{(n)} &= A_1S^{(n)} + A_2P_{yy}^{(1)} + A_3S^{(1)} + A_4\frac{\partial v_y^{(1)}}{\partial t} \\ j^{(n)} &= B_1S^{(n)} + B_2P_{yy}^{(1)} + B_3S^{(1)} + B_4\frac{\partial v_y^{(1)}}{\partial t} \end{aligned} \tag{2.7}$$

Now we find the reflection and transmission coefficients for the entire system of layers. We denote the total thickness of all layers by H. The origin of the coordinate system is placed at the lower boundary of the first layer. We assume that the sound wave enters at an angle $\theta^{(n+1)}$ from a fluid medium at $(n+1)$. The expression for the total sound potential of the incident and the reflected waves in this medium has the form

$$\varphi^{(n+1)} = [\varphi' e^{i\nu(y-H)} + \varphi'' e^{-i\nu(y-H)}]\, e^{i(\sigma x-\omega t)}, \qquad \nu = \sqrt{(k^{(n+1)})^2 - \sigma^2}$$

In the fluid on the other side of the layer system there will be only the previous sound wave

$$\varphi^{(1)} = \varphi''' e^{i\chi y} e^{i(\sigma x-\omega t)}, \qquad \chi = \sqrt{(k^{(1)})^2 - \sigma^2}$$

Using the formulas

$$\frac{\partial v_y}{\partial t} = \frac{\partial \varphi}{\partial y}, \qquad P = i\rho\omega\varphi$$

which hold for a fluid medium, and taking into account (2.6), we obtain

$$\begin{aligned} P_{yy}^{(n)} &= -i\,(1 - m^{(n)})\,\rho^{(n+1)}\omega\,(\varphi' + \varphi''), \qquad & P_{yy}^{(1)} &= -i\,(1 - m^{(2)})\,\rho^{(1)}\omega\varphi''' \\ S^{(n)} &= -\,i m^{(n)}\rho^{(n+1)}\omega\,(\varphi' + \varphi''), & S^{(1)} &= -\,i m^{(2)}\rho^{(1)}\omega\varphi''' \\ j^{(n)} &= i\nu\,(\varphi' - \varphi''), & \partial v_y^{(1)} / \partial t &= i\chi\varphi''' \end{aligned} \qquad (2.8)$$

(the general factor $e^{i(\sigma x-\omega t)}$ is omitted for brevity).

By substituting (2.8) into (2.7) we obtain two equations for the determination of the reflection and transmission coefficients. From these we find

$$W = \frac{\varphi''}{\varphi'} = \frac{Z - Z^{(n+1)}}{Z + Z^{(n+1)}}$$

$$D = \frac{\rho^{(1)}\varphi'''}{\rho^{(n+1)}\varphi'} = \frac{2\,(1 - m^{(n)} - m^{(n)}A_1)}{[(1 - m^{(2)})\,A_2 + m^{(2)}A_3]\,Z^{(1)} - A_4}\,\frac{ZZ^{(1)}}{Z + Z^{(n+1)}} \qquad (2.9)$$

Here

$$\frac{1}{Z} = -\,(1 - m^{(n)} - m^{(n)}A_1)\,\frac{[(1 - m^{(2)})\,B_2 + m^{(2)}B_3]\,Z^{(1)} - B_4}{[(1 - m^{(2)})\,A_2 + m^{(2)}A_3]\,Z^{(1)} - A_4} - m^{(n)}B_1$$

$$\left(Z^{(1)} = \frac{\rho^{(1)}\omega}{\chi}, \quad Z^{(n+1)} = \frac{\rho^{(n+1)}\omega}{\nu}\right)$$

The quantity Z can be treated as an entrance impedance of the layer system, $Z^{(1)}$ and $Z^{(n+1)}$ as the impedances of the fluid in the first and the $(n+1)$st layers, respectively.

3. Reflection from a porous layer. Let us study now the reflection of a sound wave from a single porous layer 2, separating two fluid media *1* and *3*, Fig. 2.

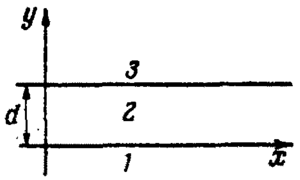

Fig. 2.

We assume, as above, that the wave is incident onto the layer from the upper medium. The reflection and transmission coefficients can be found from the general formulas (2.9), where now one should use $n = 2$. In this case, matrix Π coincides with matrix C. If we limit ourselves to the case of normal wave incidence then we find the following values for the coefficients $A_1, \ldots, A_4, B_1, \ldots, B_4$:

$$A_1 = \frac{1}{N(Q+RM_2)}(K_1\Gamma_2 r \sin P_1 - K_2\Gamma_1 \sin P_2)$$

$$A_2 = \frac{1}{N}(L_{12}\Gamma_2 r \sin P_1 \cos P_2 - \Gamma_1 \sin P_2 \cos P_1)$$

$$A_3 = -\frac{1}{N(Q+RM_2)}(K_1\Gamma_2 r \sin P_1 \cos P_1 - K_2\Gamma_1 \sin P_2 \cos P_1)$$

$$A_4 = \frac{i}{Nc_1}K_{12} \sin P_1 \sin P_2, \qquad B_1 = \frac{ic_2\Gamma_1\Gamma_2}{N(Q+RM_2)}(\cos P_1 - \cos P_2)$$

$$B_2 = \frac{ic_1}{NK_{12}}[(\Gamma_1^2 + L_{12}^2\Gamma_2^2 r^2)\sin P_1 \sin P_2 - 2rL_{12}\Gamma_1\Gamma_2(1 - \cos P_1 \cos P_2)]$$

$$B_3 = -\frac{ic_1}{N(Q+RM_2)K_{12}}[(K_2\Gamma_1^2 + K_1\Gamma_2^2 L_{12} r^2)\sin P_1 \sin P_2 - r\Gamma_1\Gamma_2(K_1 + L_{12}K_2)(1 - \cos P_1 \cos P_2)]$$

$$B_4 = A_2, \qquad N = L_{12}\Gamma_2 r \sin P_1 - \Gamma_1 \sin P_2 \quad r = \frac{c_2}{c_1}$$

$$K_1 = \lambda + 2\mu + QM_1, \quad K_2 = \lambda + 2\mu + QM_2, \quad K_{12} = K_1 - L_{12}K_2$$

$$\Gamma_1 = 1 - m + mM_1, \; \Gamma_2 = 1 - m + mM_2, \; P_1 = k_1 d, \; P_2 = k_2 d$$

Here c_1 and c_2 are the velocities of the longitudinal waves of the first and second type.

The entrance impedance of the layer and the transmission coefficient take the form

$$Z = \frac{\vartheta}{\theta}, \quad D = \frac{1}{\vartheta}\left[\frac{1}{Z_1}\sin P_2 + \frac{1}{Z_2}\sin P_2\right]\frac{2Z}{Z + Z^{(3)}}$$

$$\vartheta = \frac{1}{Z_1}\sin P_2 \cos P_1 + \frac{1}{Z_2}\sin P_1 \cos P_2 + \frac{i}{Z^{(1)}}\sin P_1 \sin P_2$$

$$\theta = i\left(\frac{1}{Z_1^2} + \frac{1}{Z_1^2}\right)\sin P_1 \sin P_2 + \frac{2i}{Z_1 Z_2}(1 - \cos P_1 \cos P_2) + \frac{1}{Z^{(1)}}\left(\frac{1}{Z_1}\sin P_2 \cos P_1 + \frac{1}{Z_2}\sin P_1 \cos P_2\right) \tag{3.1}$$

where

$$\frac{1}{Z_1} = \frac{\Gamma_1 c_1}{K_{12}}\left(1 - m - \frac{mK_2}{Q + RM_2}\right), \quad \frac{1}{Z_2} = \frac{\Gamma_2 c_2}{K_{12}}\left[-(1 - m) L_{12} + \frac{mK_1}{Q + RM_2}\right]$$

The quantities Z_1 and Z_2 can be regarded as "effective" impedances of the skeleton and the fluid in the pores. If we neglect the viscosity of the fluid ($\gamma = 0$) and the coupling coefficients ($\gamma_{12} = 0$, $\sigma_{12} = 0$), then we obtain

$$Z_1 = \frac{\rho_s c_{10}}{1 - m}, \qquad Z_2 = \frac{\rho_f c_{20}}{m}$$

(c_{10} and c_{20} are sound velocities in the skeleton and in the fluid), i.e. Z_1 and Z_2 go over into the wave impedances of the elastic and the fluid components for uncoupled vibrations. The expression for the impedance of the separation boundary of a fluid and a half-space filled with a porous medium can be obtained from (3.1) with the aid of a limiting process, which corresponds to an unbounded increase of the thickness of the layer in the negative direction of the y-axis ($d \to -\infty$). Since

$$\sin k_1 d = \sin (a_1 d) \cosh (b_1 d) + i \cos (a_1 d) \sinh (b_1 d), \qquad k_1 = a_1 + ib_1$$
$$\cos k_1 d = \cos (a_1 d) \cosh (b_1 d) - i \sin (a_1 d) \sinh (b_1 d),$$

then as $d \to -\infty$ $\sin P_1$ and $\cos P_1$ grow without limit, at which time the equation $\cos P_1 = i \sin P_1$ is satisfied, and analogously $\cos P_2 = i \sin P_2$. Formula (3.1) then yields

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} \tag{3.2}$$

i.e. the impedance of the boundary of a half-space filled with a porous medium will be the result of a "parallel" connection of the "effective" impedances of the elastic and the fluid components of the medium.

Let us study the case of complete reflection from a porous layer of a sound wave with normal incidence. This case takes place under the condition $D = 0$ which, according to (3.1), leads to the equation

$$Z_1 \sin P_1 + Z_2 \sin P_2 = 0 \tag{3.3}$$

By neglecting the viscosity of the fluid in the pores and solving this equation for the values of the parameters of the medium given in [4]

$$\sigma_{11} = 0.610, \quad \sigma_{22} = 0.305, \quad \sigma_{12} = 0.043, \quad \gamma_{11} = 0.500, \quad \gamma_{22} = 0.500, \quad \gamma_{12} = 0 \tag{3.4}$$

we obtain the following values for the thicknesses of the sound-transmitting layers of different porosity:

$m = 0.5$	0.6	0.7	0.8
$d = 0.40\lambda_1$	$0.43\lambda_1$	$0.46\lambda_1$	$0.48\lambda_1$

Here λ_1 is the length of the longitudinal wave of the first type.

Assume now that on both sides of the layer there is the same fluid ($Z^{(1)} = Z^{(3)}$). From the condition $W = 0$ we obtain the equation for the determination of the thickness of a completely "transparent" layer $Z - Z^{(1)} = 0$ which, when (3.1) is considered, reduces to

$$\left[\frac{1}{Z_1^2} + \frac{1}{Z_2^2} - \frac{1}{(Z^{(1)})^2}\right]\sin P_1 \sin P_2 + \frac{2}{Z_1 Z_2}(1 - \cos P_1 \cos P_2) = 0 \qquad (3.5)$$

If $Z^{(1)}$ satisfies the equation

$$\frac{1}{Z^{(1)}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

Equation (3.5) will yield $\cos(P_2 - P_1) = 1$, and consequently

$$d = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}n \qquad (n = 1, 2, 3, \ldots)$$

where λ_2 is the length of the longitudinal wave of the second type. For the values of the parameters given in (3.4) we shall have $d = 2.3\,\lambda_1 n$. It is not difficult to see that the reflection and transmission coefficients satisfy the obvious equation

$$|W|^2 + |D|^2 = 1$$

which represents the law of conservation of energy. If the porosity of the layer approaches zero and we assume that then $Z_2 \to \infty$, we obtain on the basis of (3.1)

$$W = \frac{(Z^{(1)} - Z^{(3)})Z_1 + i(Z_1^2 - Z^{(1)}Z^{(3)})\tan P_1}{(Z^{(1)} + Z^{(3)})Z_1 + i(Z_1^2 + Z^{(1)}Z^{(3)})\tan P_1}, \qquad Z = \frac{Z^{(1)} + iZ_1 \tan P_1}{Z_1 + iZ^{(1)}\tan P_1} Z_1$$

$$D = \frac{2Z^{(1)}Z_1 \sec P_1}{(Z^{(1)} + Z^{(3)})Z_1 + i(Z_1^2 + Z^{(1)}Z^{(3)})\tan P_1}$$

which coincides with the known result for the case of a normal incidence of a wave onto a continuous elastic medium [9].

In this limiting case Equation (3.5) takes on the form

$$\sin P_1 = 0$$

From this the thickness of the "transparent" one-component layer is

$$d = {}^1/_2\lambda_1 n$$

i.e. equal to an integral multiple of the half-wave.

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